

THE TRANSPORTATION PROBLEM

Question 1

Will the initial solution for a minimization problem obtained by Vogel's Approximation Method and the Least Cost Method be the same ? Why?
(4 Marks) (May 2011)

Answer

The initial solution need not be the same under both methods.

Vogel's Approximation Method uses the differences between the minimum and the next minimum costs for each row and column.

This is the penalty or opportunity cost of not utilising the next best alternative. The highest penalty is given the 1st preference. This need not be the lowest cost.

For example if a row has minimum cost as 3, and the next minimum as 2, penalty is 1; whereas if another row has minimum 4 and next minimum 6, penalty is 2, and this row is given preference. But least cost given preference to the lowest cost cell, irrespective of the next cost.

Vogel's Approximation Method will to result in a more optimal solution than least cost.

They will be the same only when the maximum penalty and the minimum cost coincide

Question 2

How do you know whether an alternative solution exists for a transportation problem?
(4 Marks)(Nov., 2009)

Answer

The Δ_{ij} matrix = $\Delta_{ij} = C_{ij} - (u_i + v_j)$

Where c_{ij} is the cost matrix and $(u_i + v_j)$ is the cell evaluation matrix for allocated cell.

The Δ_{ij} matrix has one or more 'Zero' elements, indicating that, if that cell is brought into the solution, the optional cost will not change though the allocation changes.

Thus, a 'Zero' element in the Δ_{ij} matrix reveals the possibility of an alternative solution.

Question 3

What do you mean by Degeneracy in transportation problem? How this can be solved?

(4 Marks)

Answer

In a transportation problem, if the no of occupied cells is less than $m+n+1$, such a solution, in transportation problem, is called as degeneracy. Degeneracy can occur two ways :

- The initial basic solution can turn out to be a degenerate solution. Or
- an improved solution can turn out to be a degenerate solution

This can be solved by introducing an infinitesimally small allocation ϵ (epsilon) to least cost empty cell .so that the total number of allocated cells is equal to $m+n+1$ independent cells.

Question 4

The following matrix is a minimization problem for transportation cost. The unit transportation costs are given at the right hand corners of the cells and the D_{ij} values are encircled.

	D1	D2	D3	Supply
F ₁	3	4	4	500
F ₂	9 (88)	6 300	7 (88)	300
F ₃	4 (88)	6 (88)	5 200	200
Deamnd	300	400	300	1000

Find the optimum solution (s) and the minimum cost.

(5 Marks) (May 2011)

Answer

Δ_{ij} values are given for unallocated cells. Hece, no. of allocated cells = 5, which = $3 + 3 - 1$ = no. of columns + no of rows – 1.

Allocating in other than Δ_{ij} cells.

Factory	S1	D2	D3	Supply
	300	100	100	500
	300	300	300	300
	300	400	300	1000

This solution is optional since Δ_{ij} are non-ve. For the other optional solution, which exists since $\Delta_{ij} = 0$ at $R_3 C_1$, this cell should be brought in with a loop : $R_3, C_1 - R_1 C_1 - R_1 C_3 - R_3 C_3$.

Working Notes:

- Step I : $R_1 C_1$ (Minimum of 300, 500)
 Step II : $R_2 C_2$ (Minimum of 300, 400)
 Step III : $R_1 C_2$ balance of C_2 total : 100, R_1 Total = 100
 Step IV : $R_1 C_3$ 100 (balance of C_3 total = 200)
 Step V : $R_3 C_3$ 200

Solution I

-100			+100
	300	100	100
		300	
+100	(88)		200

Solution II

200	100	200
	300	
100		100

	Solution I	Solution II
Cost:	3 x 300 = 900	3 x 200 = 600
	4 x 100 = 400	4 x 100 = 400
	4 x 100 = 400	4 x 200 = 800
	6 x 300 = 1800	6 x 300 = 1800
	5 x 200 = 1000	5 x 100 = 500
		4 x 100 = <u>400</u>
Minimum Cost	<u>4500</u>	<u>4500</u>

Question 5

The cost per unit of transporting goods from the factories X, Y, Z to destinations. A, B and C, and the quantities demanded and supplied are tabulated below. As the company is working out the optimum logistics, the Govt.; has announced a fall in oil prices. The revised unit costs are exactly half the costs given in the table. You are required to evaluate the minimum transportation cost. (6 Marks)(June, 2009)

Destinations Factories	A	B	C	Supply
X	15	9	6	10
Y	21	12	6	10
Z	6	18	9	10
Demand	10	10	10	30

Answer

The problem may be treated as an assignment problem. The solution will be the same even if prices are halved. Only at the last stage, calculate the minimum cost and divide it by 2 to account for fall in oil prices.

	A	B	C
X	15	9	6
Y	21	12	6
Z	6	18	9

Subtracting Row minimum, we get

	A	B	C
X	9	3	0
Y	15	6	0
Z	0	12	3

A	B	C
15	0	0
0	3	0
0	9	3

Subtracting Column minimum,

No of lines required to cut Zeros = 3

		Cost / u	Units	Cost	Revised Cost
Allocation:	X → B	9	10	90	45
	Y → C	6	10	60	30
	Z → A	6	10	<u>60</u>	<u>30</u>
				<u>210</u>	<u>105</u>

Minimum cost = 105 Rs.

Alternative Solution I

Least Cost Method

	A	B Step V	C	Step I Diff.	Step III
X	15	9	6	10	3
Y	21	12	5	10	6
Z	6	18	9	10	3
	10	10	10	30	
Step I Diff.	9	3	10		
Step III Diff.	Step II	3	10		

Initial Feasible solution

X – B

Y – C

Z – A

Test for optimality

No. of allocation = 3

No. of rows $m = 3$, no. of column = 3

$$m + n - 1 = 3 + 3 - 1 = 5$$

2 very small allocation are done to 2 cells of minimum costs, so that , the following table is got:

	A	B	C
X	15	(10) 9	(e) 6
Y	21	12	(10) 6
Z	(10) 6	18	(e) 9

$$m + n - 1 = 5$$

Now testing for optimality

				u_i
		9	e	0
			6	0
	6		e	0
v_j	6	9	6	

$u_i + v_j$ for unoccupied cells

	A	B	C
X	6	-	-
Y	6	9	-
Z	-	9	-

$$\text{Diff} = C_{ij} - (u_i + v_j)$$

	A	B	C
X	9	-	-
Y	15	3	-
Z	-	9	-

All $\Delta_{ij} > 0$, Hence this is the optimal solution.

	Original Costs	Reduced Costs due to Oil Price	Qty.	Cost
X – B	9	4.5	10	45
Y – C	6	3	10	30
Z – A	6	3	10	<u>30</u>
				<u>105</u>

Total cost of transportation is minimum at Rs.105

Alternative Solution II

	A	B	C		Step I Diff.	Step III Diff.
X	7.5	4.5	3	10	1.5	1.5
Y	11.5	6	3	10	3	3
Z	3	9	1.5	10	1.5	
Qty.	10	10	10	30		
Step I Diff.	4.5	1.5	0			
Step III Diff.	Step II	1.5	0			

Initial Solution :

	A	B	C
X	7.5	4.5 10	3
Y	11.5	6	3 10
Z	3 10	9	4.5

No. of rows + no. of column – 1

$$m + n - 1 = 5$$

No. of allocation = 3

Hence add 'e' to 2 least cost cells so that

X	7.5	4.5 10	3 e
Y	11.5	6	3 10
Z	3 10	9	4.5 e

Now $m + n - 1 = 5$

Testing for optimality,

u_i, v_j table

	A	B	C	u_i
X		4.5	e	0
Y			3	0
Z	3		e	0
v_j	3	4.5	3	

$u_i + v_j$ for unoccupied cells

	3	-	-
	3	4.5	-
	-	4.5	-

C_{ij}

7.5	-	-	
11.5	6	-	
-	9	-	

u_i+v_j

3	-	-	
3	4.5	-	
-	4.5	-	

$$\Delta_{ij} = C_{ij} - (u_i + v_j)$$

4.5	-	-	
11.5	1.5	-	
8.5	4.5	-	

All $\Delta_{ij} > 0$. Hence the solution is optimal.

	Qty.	Cost/u	Total Cost
X – B	10	4.5	45
Y – C	10	3	30
Z – A	10	3	30
Total minimum cost at revised oil prices			105

Question 6

A company has three plants located at A, B and C. The production of these plants is absorbed by four distribution centres located at X, Y, W and Z. the transportation cost per unit has been shown in small cells in the following table:

Distribution Centres Factories	X	Y	W	Z	Supply (Units)
A	6	9	13	7	6000
B	6	10	11	5	6000
C	4	7	14	8	6000
Demand (Units)	4000	4000	4500	5000	18000 17500

Find the optimum solution of the transportation problem by applying Vogel's Approximation Method. (8 Marks)(Nov., 2010)

Answer

Step 1 : Initial Allocation based on Least cost cells corresponding to highest differences

	X	Y	W	Z	Dummy	Total
A		2,000	3,500		500	6,000
B			1,000	5,000		6,000
C	4,000	2,000				6,000
TOTAL	4,000	4,000	4,500	5,000	500	18,000

Step 2 : Δ_{ij} Matrix values for Unallocated cells

	X	Y	W	Z	Dummy
A	0			0	
B	2	3			2
C			3	3	2

All Δ_{ij} values ≥ 0 . Therefore initial allocation is optimal.

Step 3 : Optimal Transportation Cost

	Units	Costs involved	Total
A to Y	2,000	9	18,000
A to W	3,500	13	45,500
B to W	1,000	11	11,000
B to Z	5,000	5	25,000
C to X	4,000	4	16,000
C to Y	2,000	7	14,000
Total minimum cost			1,29,500

Note : Since there are zeroes in the Δ_{ij} Matrix alternate solutions exist.